Exponential Notation

INTRODUCTION

Chemistry as a science deals with the qualitative and quantitative aspects of substances. In the qualitative part, we deal with the general and specific properties of substances. In the quantitative part, we are concerned with the quantities involved. This quantitative part necessitates a thorough and working knowledge of basic mathematical skills.

In this first unit, we will deal with exponential notation, because chemistry deals, oftentimes, with numbers that are infinitesimally small as well as those that are incomprehensibly large.

The use of exponential notation allows us to work with these numbers successfully and efficiently.

OBJECTIVES

1. The student will be able to correctly express quantities in exponential notation as well as convert from exponential notation to the quantity, correctly using the methods described in this unit.

2. The student will be able to add, subtract, multiply, and divide numbers expressed in exponential notation, as described in this unit.

DISCUSSION

A. Exponents

An exponent is a number that is used to indicate the number of times another number is to be multiplied by itself.

An exponent (or power) is written as a superscript to the right of the number.

Following are some examples of numbers raised to powers or exponents.

\[ 2^2 \text{ Exponent } = 2 \]
\[ 3^4 \text{ Exponent } = 4 \]
\[ 10^6 \text{ Exponent } = 6 \]

If we perform the operation that is indicated by the exponent, we would do the following and obtain the results given.

\[ 2^2 = 2 \times 2 = 4. \]
\[ 3^4 = 3 \times 3 \times 3 \times 3 = 81. \]
\[ 10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000. \]

In the above examples, the exponents (2, 4, 6) were applied to different numbers (2, 3, 10). These numbers are called the base numbers. For the operations in chemistry, we can express all the numbers as having a base of ten.
Let us review the “powers of ten” and write them in a slightly different fashion.

\[
\begin{align*}
10^0 &= 1.0 \times 10^0 = 1.0 \\
10^1 &= 1.0 \times 10^1 = 10 \\
10^2 &= 1.0 \times 10^2 = 100 \\
10^3 &= 1.0 \times 10^3 = 1,000 \\
10^4 &= 1.0 \times 10^4 = 10,000 \\
10^5 &= 1.0 \times 10^5 = 100,000 \\
10^6 &= 1.0 \times 10^6 = 1,000,000
\end{align*}
\]

The numbers in the middle column are the powers of ten written in exponential notation. This is the fashion that we will use.

B. Numbers Greater than Zero

For those numbers we wish to express in exponential notation that are greater than zero, we will use the following system.

1. Move the decimal place to the left until there is only one integer (number) to the left of it.
2. Write the number, but eliminate any zeros between the number and the original decimal place.
3. Multiply this number by ten raised to the power that is equal to the number of decimal places moved.

**Example Problem (1)**

Write 325,000 in exponential notation.

**Solution**

\[
325,000 = 3.25 \times 10^5
\]

**Example Problem (2)**

Express 7,475,000 in exponential notation.

**Solution**

\[
7,475,000 = 7.475 \times 10^6
\]

In order to convert a number from exponential notation, the following process is used.

1. Write the number but leave off the exponential portion.
2. Move the decimal place to the right the number of places indicated by the exponent.
Example Problem (3)

Change $5.24 \times 10^9$ from exponential notation.

Solution

$$5.240000000 = 5,240,000,000$$

Clearly, it is much simpler to write:

$$6.023 \times 10^{23}$$

than

$$602300000000000000000000.$$!!

C. Numbers Less than Zero

For those numbers we wish to express that are less than zero, we will use the following system.

1. Move the decimal place to the right until there is one integer to the left of the decimal.

2. Write the number, but eliminate any zeros between the number and the original decimal place.

3. Multiply this number by ten raised to the negative of the number that is equal to the number of places the decimal has been moved.

Example Problem (4)

Express .00375 in exponential notation.

Solution

$$0.00375 = 3.75 \times 10^{-3}$$

Example Problem (5)

Express .00000005075 in exponential notation.

Solution

$$0.00000005075 = 5.075 \times 10^{-7}$$

In order to convert a number from exponential notation, the following process will be used.

1. Write the number but leave off the exponential portion.

2. Move the decimal place to the left the number of places indicated by the exponent.
Example Problem (6)

Change 6.305 x 10^{-5} from exponential notation.

Solution

\[ .0000635 \]

D. Addition and Subtraction

The basic arithmetic operations can be performed on numbers expressed in exponential notation, as long as the exponents of the numbers are the same.

In some cases, this will involve changing the exponents of the numbers.

The following examples will illustrate the operation.

Example Problem (7)

Add the numbers 2.075 x 10^2 + 5.2916 x 10^3

Solution

The numbers may be added as soon as both of the exponents are the same.

Let us change the exponent 2 in the first number to 3.

\[ 2.075 \times 10^2 = .2075 \times 10^3 \]

To do this, the decimal in the original number was moved to the left. The number of places needed was determined by the change needed in the exponent. In this case, the change was one, therefore, one place to the left.

The two numbers may now be added.

\[ \begin{align*}
.2075 \times 10^3 \\
+ 5.2916 \times 10^2 \\
\hline
5.4991 \times 10^3
\end{align*} \]

The exponent of the sum of the two numbers is the same as the exponent of the two numbers.

Exponent Problem (8)

What is the difference between 7.025 x 10^{-6} and 2.31 x 10^{-7} ?
Solution

The difference between these numbers may be found as soon as they are both expressed with the same exponent.

Let us change the exponent of minus seven in the second number to a minus six.

\[2.31 \times 10^{-7} = 0.231 \times 10^{6}\]

To do this, we move the decimal in the number one place to the left and add one to the exponent, changing from \(-7\) to \(-6\).

The difference of the two numbers may now be found.

\[
\begin{array}{c}
7.025 \times 10^{-6} \\
- 0.231 \times 10^{-6} \\
\hline
6.794 \times 10^{-6}
\end{array}
\]

The exponent of the difference of the two numbers is the same as the exponent of the two numbers.

E. Multiplication and Division

The most common operations in most calculations are multiplication and division.

In multiplication of numbers written in exponential notation, the exponents are added and the numbers multiplied.

In division, the exponent of the number in the denominator is subtracted from the exponent of the number in the numerator.

Example Problem (9)

Multiply \(2.1 \times 10^3 \times 1.2 \times 10^{-2}\)

Solution

In multiplication, no change in exponents is necessary, as was the case with addition and subtraction.

\[
\begin{array}{c}
2.1 \times 10^3 \\
\times 1.2 \times 10^{-2} \\
\hline
2.52 \times 10^1
\end{array}
\]

The numbers were simply multiplied and the exponents added.

\[3 + (-2) = 1\]

Example Problem (10)

Divide \(6.2 \times 10^{-2}\) by \(2.0 \times 10^5\)
Solution

In division, as in multiplication, the exponents need not be changed.

\[
\frac{6.2 \times 10^{-2}}{2.0 \times 10^{5}} = 3.1 \times 10^{-7}
\]

The numbers were simply divided and the difference in exponents found between numerator and denominator.

\[-2 - (+ 5) = -7\]
PROBLEMS

1. Express the following in exponential notation.
   a. 7,250,000.
   b. 752.
   c. .001079
   d. .00000635
   e. 62.48

2. Convert the following from exponential notation.
   a. 6.23 x 10^{10}
   b. 1.85 x 10^{-4}
   c. 6.035 x 10^{-11}
   d. 5.25 x 10^{6}
   e. 6.0009 x 10^{-3}

3. Perform the following additions and subtractions.
   a. \[ 6.03 \times 10^3 + 2.15 \times 10^5 \]
   b. \[ 1.19 \times 10^{-2} - 2.1 \times 10^{-4} \]
   c. \[ 8.39 \times 10^4 + 2.17 \times 10^{-1} \]
   d. \[ 6.48 \times 10^{-11} - 7.256 \times 10^{-9} \]

4. Perform the following multiplications and divisions.
   a. \[ 2.37 \times 10^{-11} \times 1.89 \times 10^{-9} \]
   b. \[ 8.64 \times 10^{12} \times 2.16 \times 10^{-12} \]
   c. \[ \frac{3.3642 \times 10^{-11}}{8.129 \times 10^{21}} \]
   d. \[ \frac{1.982 \times 10^{10}}{2.192 \times 10^{-2}} \]